ANALYSIS OF THREE NARROW TRANSVERSE STRIPS IN WAVEGUIDE

Kai Chang
Shared Applications, Inc.
611 Church Street
Ann Arbor, Michigan 48104
U. S. A.

Peter J. Khan
Electrical Engineering Department
University of Queensland
St. Lucia, Queensland 4067
Australia

ABSTRAT

A theoretical determination with experimental verification is made for three narrow transverse strips of different widths, unsymmetrically located in a rectangular waveguide. The analysis is based on extremization of current density ratios among the strips.

Introduction

This paper reports an analysis of three narrow, transverse, inductive strips of different widths, unsymmetrically located in a rectangular waveguide. The current density ratios among the strips are obtained by extremization of the variational form, and solution of the two unknowns in the resulting two nonlinear equations through generalization of the half-section numerical method.

Previous studies by Craven and Lewin[1] were confined to three small-diameter vertical posts evenly-spaced across the waveguide transverse plane; a similar geometry was considered by Mariani[2]. Recently Lewin[3,4] has developed an analytical technique applicable to the general unsymmetrical multiple-strip geometry through extension of the Carleman equation to multiple intervals. The present formulation, which extends the two-strip variational method used by Chang and Khan[5], has the advantage of relative mathematical simplicity.

The structure considered here has considerable potential application in microwave tuning and filter networks.

Theoretical Analysis

The structure is shown in Fig. 1. Three strips of unequal widths placed unsymmetrically in a rectangular waveguide at the plane $z=0$.

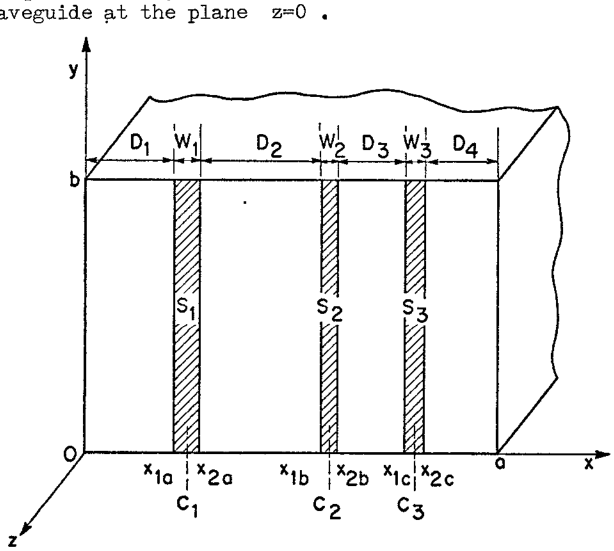

Fig. 1 : Cross section of a rectangular waveguide with three infinitesimally-thin strips in the same transverse plane.

The incident dominant-mode electric field is given by :

$$\underset{\sim}{E}_i = \sin\left(\frac{\pi x}{a}\right) \exp\left(-\Gamma_1 z\right) \underset{\sim}{y} . \tag{1}$$

For a single strip Collin[6] shows that the normalized shunt susceptance $\bar{B}_T$ may be expressed in the following stationary form :

$$\bar{B}_T = \frac{-2 \left\{ \int_S J_y(x,y) \sin\left(\frac{\pi x}{a}\right) dx\, dy \right\}^2}{\tau_1 \sum_{n=2}^{\infty} \frac{1}{\Gamma_n} \left\{ \int_S J_y(x,y) \sin\left(\frac{n\pi x}{a}\right) dx\, dy \right\}^2} \tag{2}$$

where $\Gamma_n = \left(\frac{n^2\pi^2}{a^2} - k_0^2\right)^{\frac{1}{2}}$ and $\tau_1 = -j\Gamma_1$.

$J_y(x,y)$ is the y-directed current density in the strip of surface S .

For three strips we may take S to be surfaces $S_1 + S_2 + S_3$ and assume the current density $J_y(x,y)$ for the narrow strips in the form :

$$\begin{aligned} J_y(x,y) = A &\left\{ U(x-x_{1a}) - U(x-x_{2a}) \right\} \\ &+ f_1 A \left\{ U(x-x_{1b}) - U(x-x_{2b}) \right\} \\ &+ f_2 A \left\{ U(x-x_{1c}) - U(x-x_{2c}) \right\} \end{aligned} \tag{3}$$

where A is an amplitude constant and $U(x)$ is the unit step function.

Substituting Eq. (3) into Eq. (2) and integrating over S , we obtain :

$$\bar{B}_T = -\frac{2}{\tau_1} \frac{E_T^2}{\sum_{n=2}^{\infty} H_{nT}^2} \tag{4}$$

where

$$E_T = E_a + f_1 E_b + f_2 E_c$$

$$H_{nT} = H_{na} + f_1 H_{nb} + f_2 H_{nc}$$

and

$$E_i = \cos\left(\frac{\pi x_{1i}}{a}\right) - \cos\left(\frac{\pi x_{2i}}{a}\right) \text{ for } i = a, b, c$$

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$$H_{ni} = \frac{1}{n\sqrt{\Gamma_n}} \left\{ \cos\left(\frac{n\pi x_{1i}}{a}\right) - \cos\left(\frac{n\pi x_{2i}}{a}\right) \right\}$$

for $i = a, b, c.$

To find f_1 and f_2 , we put $\frac{\partial \bar{B}_T}{\partial f_1} = 0$ and $\frac{\partial \bar{B}_T}{\partial f_2} = 0$.
Thus, the following equations are obtained :

$$E_T \left\{ E_T \sum_{n=2}^{\infty} H_{nj} H_{nT} - E_j \sum_{n=2}^{\infty} H_{nT}^2 \right\} = 0 \qquad \text{for } j = b, c. \qquad (5)$$

The set of two equations in (5) is solved by an extended form of the half-section numerical technique.

The analysis can be extended to capacitive strips[7] by assuming suitable current distribution in Eq. (3).

Comparison with Experimental Measurements

Measurements were carried out with strips in conventional X-band waveguide having $a = 0.900$ in. and $b = 0.400$ in. Results shown in Fig. 2, as a function of frequency, indicate close agreement with the theory. Also shown is the sum of $\bar{B}_1$, $\bar{B}_2$ and $\bar{B}_3$ which are isolated single-strip susceptances found from Eq. (2) using the conventional constant-current assumption. It is interesting to note $\bar{B}_T / \lambda_g$ is almost independent of frequency, as found by Lewin for the multi-aperture obstacle[3]. Fig. 3 shows the current ratio of the three-strip obstacle.

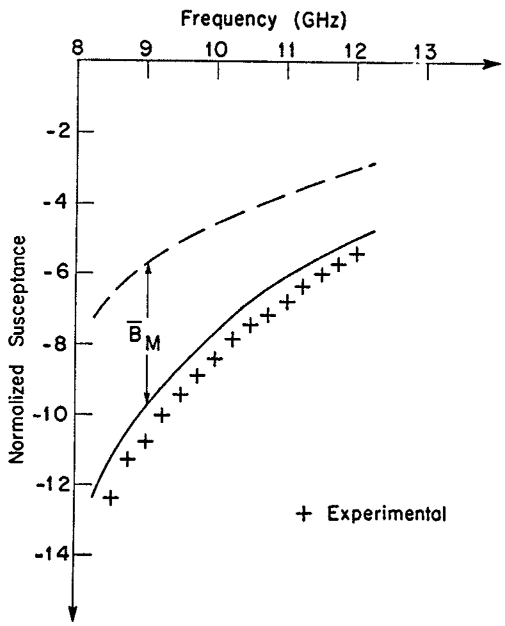

Fig. 2 : Susceptance of the three-strip obstacle. The solid line shows the susceptance value $\bar{B}_T$; the broken line shows the sum $\bar{B}_1 + \bar{B}_2 + \bar{B}_3$.
$C_1 = 0.3945$ in., $C_2 = 0.4945$ in., $C_3 = 0.6945$ in.; $W_1 = 0.0770$ in., $W_2 = 0.0830$ in., $W_3 = 0.1190$ in.; $D_2 = 0.0200$ in., $D_3 = 0.1000$ in.

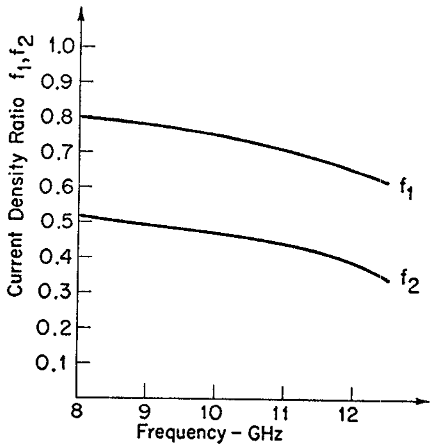

Fig. 3 : The current density ratio of the three-strip obstacle with dimensions shown in Fig. 2 .

For three strips of the same width, evenly spaced across the waveguide transverse plane, the solution to Eq. (5) gives $f_1 = \sqrt{2}$ and $f_2 = 1$, in agreement with the results of Craven and Lewin[1] for small-diameter posts. The comparison is shown in Fig. 4 .

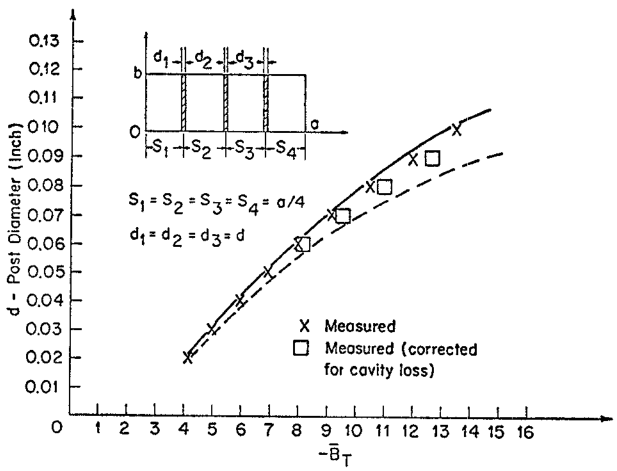

Fig. 4 : Normalized susceptance of three-post obstacle. The solid line shows our theoretical results; the broken line shows Craven and Lewin's theoretical results. Frequency is at 4 GHz.; $a = 2$ in. and $b = 2/3$ in.

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